

Next one determines the shock temperature  $T_D$  at this same volume by means of the relation

$$T_D - T_s = \frac{\Delta e_D - \Delta e_s}{C_v} \quad (14)$$

The numerator of Eq. (14) may be evaluated by means of Eqs. (3) and (4) which, with the help of Eq. (6), may be put in the form

$$\Delta e_D - \Delta e_s = \frac{(\frac{1}{2})p_s(v_0 - v) - \int_v^{v_0} p dv}{1 - \frac{v_0 - v}{2C_v} \left( \frac{\partial p}{\partial T} \right)_v} \quad (15)$$

Once  $T_D$  and  $T_s$  are known, it is very simple to compute  $\Delta s$ , for

$$\Delta s = C_v \ln(T_D/T_s) \quad (16)$$

It should perhaps be noted that Eqs. (14) and (16) are both based on processes occurring at constant volume, in which of course no mechanical work is performed by the system. On the other hand, the integrals in Eqs. (13) and (15) are to be evaluated at constant entropy.

It will be clear that all the corrections to be made depend on information about the temperature and volume dependence of the various thermodynamic variables. Fortunately  $C_v$  and  $(\partial p/\partial T)_v$  do not vary much under the conditions covered by the observations. The calculations for Duralumin have included an estimate of this variation, though it has been found that the final results would not be significantly changed if both quantities were taken as constants.

The necessary calculations are quite straightforward once an approximate expression for  $p$  as a function of compression at constant entropy is known. The empirical form chosen is

$$p_s = \alpha_s \mu + \beta_s \mu^2 \quad (17)$$

the entropy being constant. The value of the constant  $\alpha_s$  is inferred from known values of the velocity of sound and of the density under standard laboratory conditions. Thus the data derived from shock measurements are used merely to evaluate the constant  $\beta_s$ . Some question naturally arises as to how  $\alpha_s$  is related to the observed sound velocity. Equation (17) is intended to apply to material under such great hydrostatic pressure that any shearing stress is completely negligible both in its magnitude and in its effect on the compression  $\eta$ . Accordingly, it would seem natural to evaluate  $\alpha_s$  for conditions under which compression occurs without appreciable shearing stress. But determinations of sound velocity in general are made either with bars, for which  $c_1 = (E/\rho)^{1/2}$  where  $E$  is Young's modulus, or for large masses of material for which  $c_2 = [(k + 4G/3)/\rho]^{1/2}$  where  $k$  is the bulk modulus and  $G$  the shear modulus. It is the isentropic bulk modulus which relates pressure

to compression when the shearing stress is negligible, and accordingly we have assumed that

$$\alpha_s = (\partial p/\partial \mu)_s = -v_0 (\partial p/\partial v)_s = k_s \quad (18)$$

A value of  $\alpha_s$  may be quickly deduced from  $c_1$ , if Poisson's ratio  $\nu$  is known, or from  $c_2$  if the velocity of shear waves  $c_3 = (G/\rho)^{1/2}$  is known. In the latter case, the computation is obvious; in the former,

$$\alpha_s = E_s/3(1 - 2\nu) \quad (19)$$

Most of the currently available data on compressibilities at extreme pressures have been obtained isothermally. These data may likewise be fitted well by an equation similar to Eq. (17):

$$p_T = \alpha_T \mu + \beta_T \mu^2 \quad (20)$$

The relation between  $\alpha_s$  and  $\alpha_T$  is well known to be

$$\alpha_s - \alpha_T = (v_0 T/C_v) (\partial p/\partial T)_v^2 \quad (21)$$

The corresponding difference between  $\beta_s$  and  $\beta_T$  is not so well known, but may be written

$$\beta_s - \beta_T = -(\alpha_s - \alpha_T) \left\{ 1 - \left( \frac{1}{2} \right) v_0 \left[ (1/C_v) (\partial p/\partial T)_v + 3(\partial^2 p/\partial v \partial T)/(\partial p/\partial T)_v - 3(T/C_v) (\partial^2 p/\partial T^2)_v - (T/C_v^2) (\partial p/\partial T)_v (\partial C_v/\partial T)_v \right] \right\} \quad (22)$$

In the course of developing experimental techniques with a view to determining what ultimate precision is possible, it was found convenient to use an alloy of aluminum with superior mechanical properties rather than the pure element for which static compressibilities are available. In order to compare the present work with that of others,<sup>2</sup> it is desirable to estimate the effects of the alloying constituents. This can be done easily if one assumes that the volume of the alloy is equal to the sum of the volumes of its constituents. For many alloys this assumption leads to an excellent estimate of the normal density, and it seems reasonable to expect that its validity is not appreciably worse at high pressures. The subscripts (1) and (2) will be used to denote properties of the constituents; absence of either of these denotes a property of the alloy. The additional subscript (o) refers to a property under standard laboratory conditions. Thus the equations of state involved would be

$$\begin{aligned} p &= \alpha \mu + \beta \mu^2, \\ p &= \alpha_1 \mu_1 + \beta_1 \mu_1^2, \\ p &= \alpha_2 \mu_2 + \beta_2 \mu_2^2, \end{aligned} \quad (23)$$

while the equation connecting the various compressions is

$$\frac{1}{\rho_0(\mu + 1)} = \frac{X_1}{\rho_{01}(\mu_1 + 1)} + \frac{X_2}{\rho_{02}(\mu_2 + 1)} \quad (24)$$

$X_1$  and  $X_2$  denote the fractions by mass of the respective constituents. Values of  $\alpha$  and  $\beta$  in terms of  $\alpha_1, \alpha_2, \beta_1,$  and

<sup>2</sup> P. W. Bridgman, Proc. Am. Acad. Arts Sci. 77, 189 (1949).